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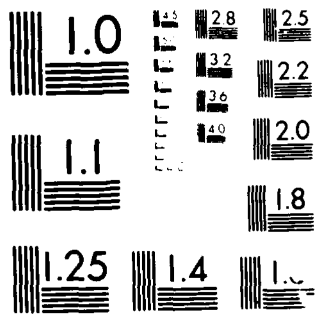


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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes the fix simulation program available at Mathematical Analysis Research Corp. (MARC, 4329 Via Padova, Claremont, CA 91711). This program simulates lines-of-bearing (LOB's) between sensors and emitters, subsequent fix estimation (with several fix algorithms available), and EEP ellipse generation. Results are displayed on a terminal screen and can be obtained in hard copy on a graphics plotter. Finally, the report describes bias results for all of the available fix algorithms.

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U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
Software Analysis and Management System

Algorithms and Intuition

EAAF

Technical Memorandum No. 9

17 April 1987

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# PREFACE

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## EXECUTIVE SUMMARY

This Technical Memorandum was prepared to satisfy the often asked question by customer personnel in the field " what are algorithms and how do they apply to my work? ". This report was requested verably by the Combat Developers Support Facility personnel during FY-86 and used FY-86 funds.

The work in this tutorial memo is supported by a simulation system which runs on the IBM PC- series of computers. This combination can be useful for training new analysts.

MARC MATHEMATICAL ANALYSIS RESEARCH CORPORATION

TO: Jim Gillis

SUBJECT: Algorithms and Intuition

DATE: 20 June 1985

INTRODUCTION

Algorithms have idiosyncrasies and behavior patterns. Study can reveal some of these phenomena. In emitter detection, location and identification, however, a set of algorithms are intertwined into a system. Reduction of behavior patterns of individual algorithms to mathematical expressions does not necessarily resolve the problem of interpreting these results in terms of system behavior. There are two policies that MARC follows to make results more relevant to analysis of system behavior:

(1) Study combinations of algorithms and algorithm interfaces
(2) Reduce algorithm behavior to an intuitive format (geometric if possible) so that qualitative analysis may be used to expand algorithmic insights into systematic insights.

The relationship between basing ellipses on the F test and using a Chi-square ellipse combination algorithm is an example where MARC has followed (1) above. It is the objective of this memo, however, to illustrate one of MARC's tools in pursuit of (2) above. In particular MARC uses a graphical LOB simulator which allows overlay of algorithm results including competing algorithms.

In Section I below the current version of the program is discussed briefly. In Section 2 the use of the LOB simulator program is demonstrated by showing some types of screens and reports available. In Section 3 there is a discussion of algorithms which provides some of the basic materials used to develop intuition regarding differences and similarities in Fix behavior. In Section 4 a summary of bias results (which will be reported more completely in another report) is used to illustrate an example of the results of a mix of analysis and intuition.

II. THE LOB AND FIX SIMULATOR PROGRAM

The main menu of this program is shown in figure 1. The options and capabilities of each menu option are discussed below. Note that current capabilities are specified although MARC frequently modifies the program for special applications.

A. Locate Sensors

This option allows one to specify up to 10 locations LOBs will be taken from. (More than 10 locations may be specified with certain limitations when ellipse combination is being used. See options I and J. The limit of 10 is often changed depending on the application and competing memory requirements.)



4239 VIA PADOVA, CLAREMONT CA 91711

```
Enter the letter of your choice:
--------------------------------
 (A) Modify sensor set-up
 (B) Move Emitter at (3,4.5)
 (C) Change or view sensor accuracy
 (D) Change FIX algorithm
       ... currently Weighted Perpendicular
 (E) Graph LOBs
 (F) Simulate Fixes
 (G) Change Save Fix Mode
       ... currently OFF
 (H) Combine Saved Fixes
 (I) Set Multiple Fix Pattern
 (J) Combine according to Pattern
 (K) Simulate Combinations
 (L) Graph Scaling: Currently AUTOMATIC
 (X) Quit
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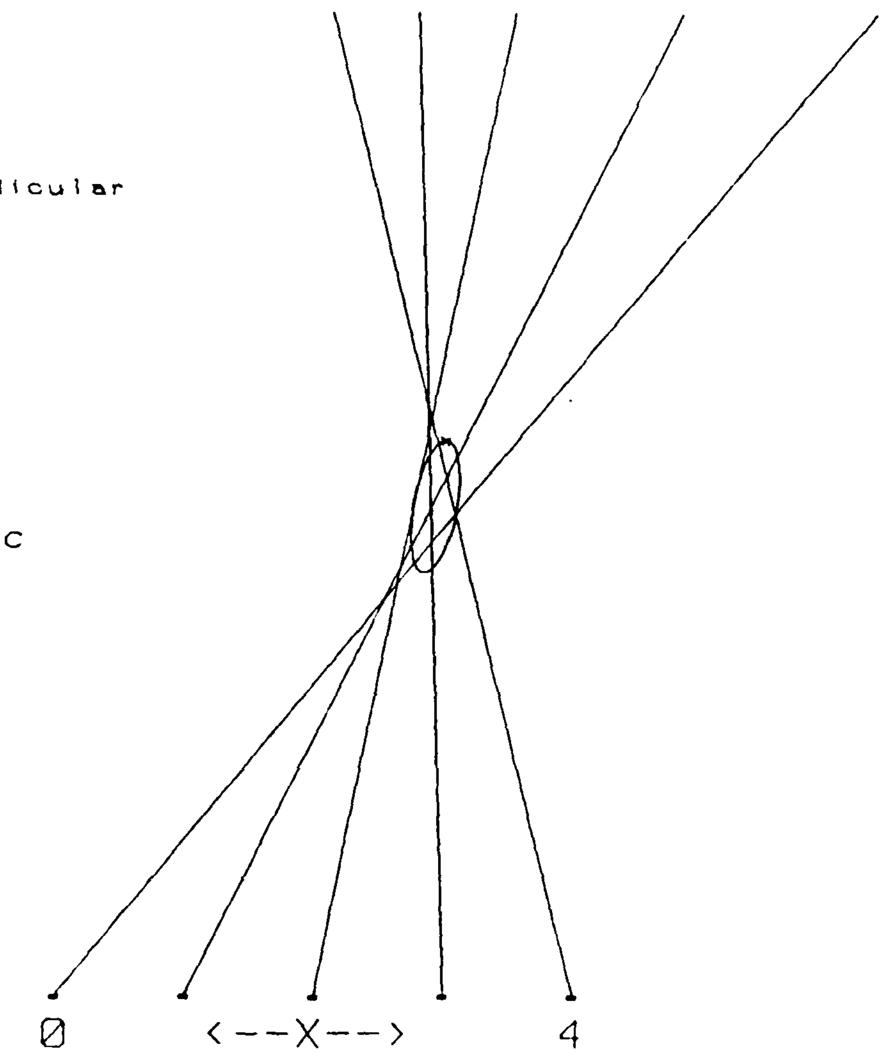


Figure 1

B. Locate the Emitter

Only one emitter may be currently specified although MARC intends to create a modification of this program that allows multiple emitters for study of wild bearings and tests for combination of data.

C. Specify Sensor Accuracy

This option allows specification of different standard deviations for the angular accuracy of individual LOBs. It also allows specification of a standard deviation for location error although it is only implemented for the study of one Fix algorithm. Recent information communicated to MARC has also suggested that a different model may be appropriate for location error.

D. Change Fix Algorithm

This option allows the fix algorithm to be

1) none (LOBs can be shown without fixes, etc.. This option has been useful for production of graphs for reports.)
2) Perpendicular
3) Weighted Perpendicular
4) Angular Error Minimization
5) Weighted Perpendicular with location error
6) F test based weighted perpendicular (the other versions use knowledge of the true accuracy and a Chi-Square distribution)

At the time this option is chosen if an LOB graph is on the screen fixes and ellipses corresponding to the new method are added to the screen except in the case of option 5) above where it would confuse the graph.

E. Graph LOBs

Random errors are generated for each LOB and the results are plotted together with a fix and ellipse if appropriate to the Fix Algorithm active. The user is then given the option of sending the results seen on the screen to a plotter.

F. Simulate Fixes

Similar to E above but only fixes are plotted and up to 5000 runs may be specified. Examples of the results of simulation may be found in Figures 2 and 3. Statistics for the simulation are also shown.

G. Change Save Fix Mode

When this is turned on all subsequent ellipses generated are combined using the standard ellipse combination method. This approach has greater flexibility than the pattern method described below under I and J.

H. Combine Saved Fixes

Actually the combination is done already and this option merely shows the result. To the User this title seemed appropriate, however.

Emitter is at ( 10 , 55 )

Perpendicular Method

---------------------

XBAR= 9.99831896486
YBAR= 56.3536212819

2 LOBs EACH

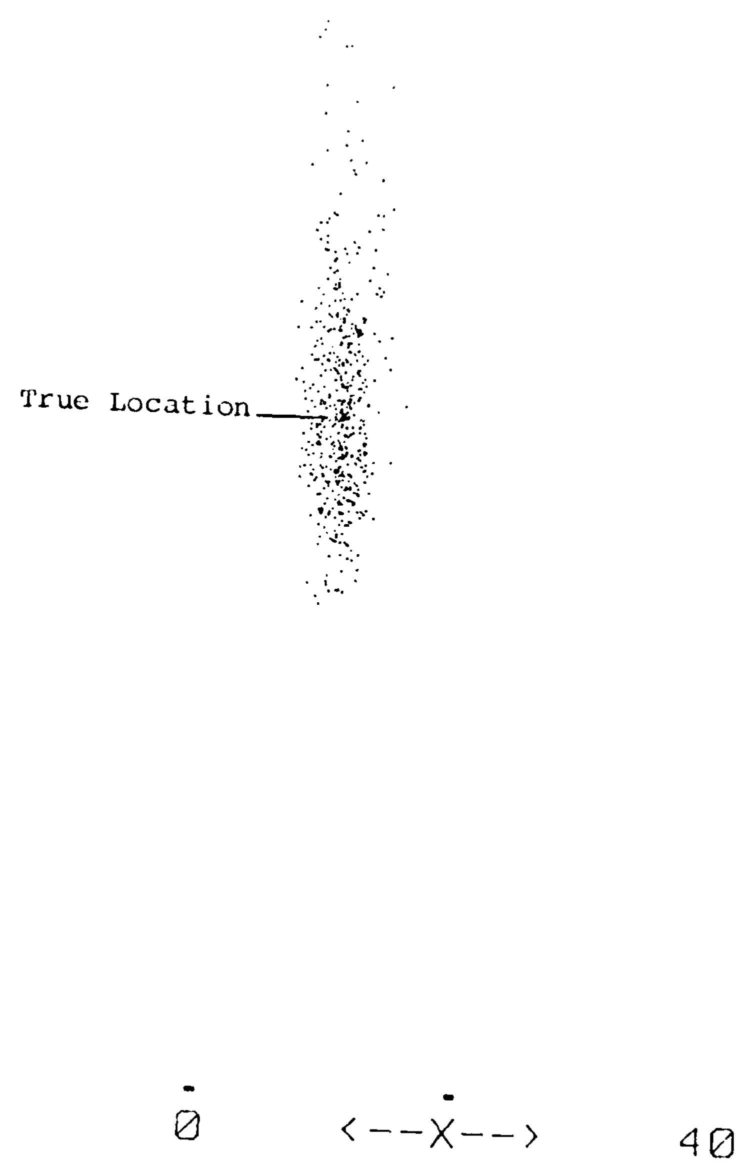


Figure 2

Emitter is at ( 10 , 55 )

Perpendicular Method

---------------------

XBAR= 9.96562431082
YBAR= 53.8402451874

10 LOBs EACH

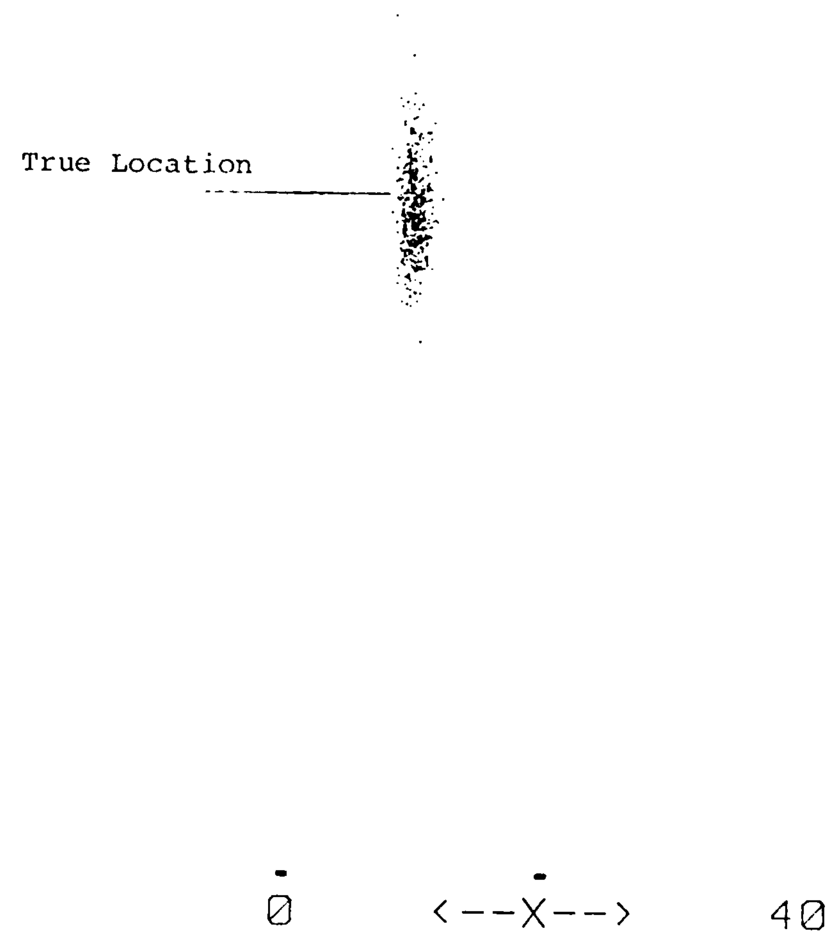


Figure 3

I. Set Multiple Fix Pattern

This is used to design a simulation of generation and combination of ellipses according to the pattern specified.

J. Combine According To Pattern

This option is redundant as it is the case of 1 simulation according to the pattern defined in I. Future versions of this program may combine this option with option K.

K. Simulate Combinations

This option simulates combinations according to a specified pattern.

L. Graph Scaling:

Allows some rescaling when the program logic does not yield the scale desired.

## III. 2-DIMENSIONAL FIX ALGORITHMS

For the purposes of this section we will assume the only source of inaccuracy is error in the direction of the bearing.

It is the intention of this section to discuss similarities and differences in the concept of various algorithms. Actual differences may be seen in the differences in the location estimates they produce. See Figure 4.

2-Dimensional Least Squares Fix Algorithms

Least Squares Algorithms are algorithms that estimate parameters by minimizing a sum of squared terms (usually a measure of error) where each term depends only on the parameters and one observation. For our application the parameters are the x and y coordinates of the location estimate and the observations are LOBs.

All but one of the algorithms to be discussed can be characterized by least squares. This is not the reason for the importance of the least squares characterizations, however. For example, all of the algorithms discussed could be characterized in defining equations for implicit functions yet this gives no insight. The least squares characterizations are important because:

(1) They allow one to analyze behavior intuitively without reference to mathematical details if necessary.

(2) Least squares is naturally related to the normal distribution and the level curves associated with it. It is these level curves that EEPs approximate.

Examples of Least Squares Algorithms:

Perpendicular Method: (The term squared is the perpendicular distance from the estimate to the LOB.)

'Intermediate Method': (The term squared is the perpendicular distance from the estimate to the LOB divided by the distance from the sensor to the location estimate. Variations involve adjusting for other terms known to affect location accuracy such as sensor accuracy.)

Emitter is at ( 5 , 7 )

Min Angular Error

----------------------

XBAR= 4.76759400962
YBAR= 6.72161868043

Perpendicular Method

----------------------

XBAR= 4.71942641906
YBAR= 6.59194583538

Weighted Perpendicular

----------------------

XBAR= 4.73255481893
YBAR= 6.626563827


Figure 4

Angular Minimization Method: (The term squared is the angle between the LOB and a line from the sensor to the location estimate.)

An Algorithm Not Definable In Terms Of Least Squares

Weighted Perpendicular Method: (This is the algorithm which seems to correspond most closely with what MARC has been exposed to in operational algorithms. It is a variation on the perpendicular method which introduces the same weights as the 'intermediate method' but using a different characterization of the perpendicular method. Although this method is not based on least squares its similarity to methods that are is a useful tool in understanding it.)

Basic Definitions (for the Least Square Methods Only):

$(X,Y)$ = true location of the emitter

$\omega_k$ = true bearing from the kth sensor to the emitter $(X,Y)$

$\theta_k$ = observed bearing from the kth sensor (multiple readings are treated as coming from different sensors)

$(x,y) = (x(\theta_1,\ldots,\theta_n), y(\theta_1,\ldots,\theta_n))$ = estimated location of the emitter

$a_k = a_k(x,y,\theta_k)$ = the 'squared error term' corresponding to the kth LOB and a location estimate of $(x,y)$ for the least square method.

$S(x,y,\theta_1,\ldots,\theta_n) = \sum_{i=1}^{n} a_k$ = the sum of squares that the method minimizes

$\sigma_k$ = standard deviation of the angular measurement of the kth LOB in radians (multiply by $\pi/180$ if in degrees)

Perpendicular Method

$$a_k = (x-\xi_k)^2\cos^2\theta_k + (y-\eta_k)^2\sin^2\theta_k - 2(x-\xi_k)(y-\eta_k)$$

Angular Minimization Method

$$a_k = [\mathrm{Arctan}((x-\xi_k)/(y-\eta_k)) - \theta_k]$$

'The Intermediate Method'

$$a_k = [(x-\xi_k)^2\cos^2\theta_k + (y-\eta_k)^2\sin^2\theta_k - 2(x-\xi_k)(y-\eta_k)]/[(x-\xi_k)^2 + (y-\eta_k)^2]$$

The 'Intermediate Method' is so close to the Angular Minimization Method that the title invented here is not really needed. (The relationship is easily shown using Taylor Series expansions.) It was included here because it makes the relationship between the Perpendicular Method and the Angular Minimization Method clearer and because it shows that the Angular Minimization Method is a type of 'weighted' Perpendicular. The method referred to in this method as the Weighted Perpendicular uses the same weights but at a different point.

## IV. BIAS

The objective of this section i. a specific pattern to differences between particular methods. In this case the differences are differences in bias behavior of the Perpendicular Method and the Angular Minimization Method.

### Formula (and notation) for Bias in the General Least Squares Case

Let subscripts on a, x or y which follow a comma denote partial derivatives. For example an 'x' subscript denotes the partial with respect to x or an i subscript denotes the partial with respect to $\theta_i$. Furthermore, let X,Y with a comma and subscript indicate the corresponding x or y term evaluated at the true location and the variables below denote the indicated term EVALUATED at the true LOBs. (Recall that Taylor Series uses evaluated derivatives.)

$$
\begin{array}{llllll}
b_k = a_{k,x} & c_k = a_{k,y} & b_k' = a_{k,kx} & c_k' = a_{k,ky} & b_k'' = a_{k,kx} & c_k'' = a_{k,ky} \\
d_k = a_{k,xx} & e_k = a_{k,xy} & f_k = a_{k,yy} & d_k' = a_{k,kxx} & e_k' = a_{k,kxy} & f_k' = a_{k,kyy} \\
g_k = a_{k,xxx} & h_k = a_{k,xxy} & i_k = a_{k,xyy} & j_k = a_{k,yyy} & &
\end{array}
$$

Computing a Taylor Series for location error in terms angular error by using implicit derivatives, taking the first order term in the series and computing its expected value yields

$$
\text{FIRST ORDER BIAS} = (-1/Q) * \begin{bmatrix} \sum_{k=1}^{n} f_k & -\sum_{k=1}^{n} e_k \\ -\sum_{k=1}^{n} e_k & \sum_{k=1}^{n} d_k \end{bmatrix} * \sum_{k=1}^{n} \begin{bmatrix} U_k \\ V_k \end{bmatrix} \sigma_k^2
$$

where $Q = (\sum_{k=1}^{n} d_k)(\sum_{k=1}^{n} f_k) - (\sum_{k=1}^{n} f_k)^2$

and $U_k = (X_{,k})^2 \sum_{m=1}^{n} g_m + 2X_{,k} Y_{,k} \sum_{m=1}^{n} h_k + (Y_{,k})^2 \sum_{m=1}^{n} i_m + 2X_{,k} d_k' + 2Y_{,k} e_k' + b_k''$

and $V_k = (X_{,k})^2 \sum_{m=1}^{n} h_m + 2X_{,k} Y_{,k} \sum_{m=1}^{n} i_k + (Y_{,k})^2 \sum_{m=1}^{n} j_m + 2X_{,k} e_k' + 2Y_{,k} f_k' + c_k''$

Implicit function theory also must be used to determine $X_{,k}$ and $Y_{,k}$. It yields

$$
\begin{bmatrix} X_{,k} \\ Y_{,k} \end{bmatrix} = (-1/Q) * \begin{bmatrix} \sum_{m=1}^{n} f_m & -\sum_{m=1}^{n} e_m \\ -\sum_{m=1}^{n} e_m & \sum_{m=1}^{n} d_m \end{bmatrix} * \begin{bmatrix} b_k' \\ c_k' \end{bmatrix}
$$

### Perpendicular Method versus Angular Minimization Method

To a first order approximation the biases turn out to be identical.

### Large numbers of LOBs

Terms that have more sums in the denominator than in the numerator go to zero (unless the terms in a sum in the denominator collapse which does not occur in our application.) As the number of LOBs becomes large our first order bias term approaches the remaining terms, namely

$$
\text{FIRST ORDER BIAS} = (-1/Q) * \begin{bmatrix} \sum_{k=1}^{n} f_k & -\sum_{k=1}^{n} e_k \\ -\sum_{k=1}^{n} e_k & \sum_{k=1}^{n} d_k \end{bmatrix} * \sum_{k=1}^{n} \begin{bmatrix} b_k'' \\ c_k'' \end{bmatrix} \sigma_k^2
$$

For angular minimization and the weighted perpendicular method this turns out to be equal to zero. Thus as the number of LOBs increases the bias goes to

zero for these two methods. For the perpendicular method this does not happen. In this case

$$b_k'' = X-\xi_k \qquad c'' = Y-\eta_k$$

Symmetric Case

Many bias effects are of different signs and hence cancel. By restricting attention to a symmetric case it is possible to show the behavior of terms less prone to cancellation. In particular, the formulation given here is for an emitter on the y-axis and such that for every LOB taken from a sensor on one side of the y-axis there is another LOB (from an equally accurate sensor) directly across the y-axis (i.e. $(\xi,\eta)$ and $(\xi,-\eta)$).

According to the predicted result for symmetric bias in the Perpendicular Method, bias changed from long to short as the number of LOBs increased. In a symmetric situation, bias in the x-direction is predicted to be zero, so bias is entirely in the y-direction. In Figures 2 and 3, only five-hundred simulations were used to generate the plotted illustrations in order to achieve a clear diagram, although more simulations would generate results closer to the predicted results. Figure 3 is identical to Figure 2 except that there are 5 sensors at each location instead of only two. Note the marked change in YBAR between the two diagrams; even five-hundred simulations showed the change of direction in bias as the number of LOBs increased from 2 to 10.

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